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Tandemat – a didactic game for secondary mathematics and its potential

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Abstract

The study concerns a mathematical game Tandemat based on the Activity game. The research questions are: What are the benefits and limits of Tandemat for the development of pupils' mathematical knowledge? What knowledge does the game bring to the teacher? Does the game have any benefits outside the field of mathematics? A qualitative analysis of experiments with five groups of pupils has shown that the game has potential to diagnose pupils' understanding, to develop and consolidate their mathematical knowledge, and to develop some non-mathematical abilities. Examples of the game playing are given to support results.

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Keywords: Games; teaching mathematics; diagnosis of pupils' knowledge; consolidation of knowledge.

1. Introduction and theoretical background

Games are used in the teaching of mathematics for a variety of purposes. There is a multitude of collections of games. Mathematical games can be used to introduce concepts, to motivate pupils, to practise skills or consolidate a concept. When playing games, pupils often feel more secure than in everyday teaching, the reason being that the games are natural for them, they usually enjoy them and in terms of mathematics, incorrect solutions are not considered as mistakes but a natural part of the game. Games provide a unique opportunity for integrating the cognitive, affective and social aspects of learning (Jančařík, 2007), and teachers often report pupils' joy and higher motivation towards learning. However, research is needed to explore the educational potential of games. Some studies are mentioned below.

Koran and McLaughlin (1990) compare games and drill in learning multiplication skills and conclude that drill should precede the game which should be used later for the maintenance of multiplication facts and as a motivator. To explore the pupils' understanding of the graphical representation of functions, a message game is used in Yavuz's (2010) study. In the game, one half of the class describe a curve to the other half who try to reproduce it as similarly as possible. While doing so, the pupils' understanding of the curves becomes visible for the researcher (and the teacher). Chow, Woodford and Maes (2011) investigated an online version of the television game show, 'Deal or No Deal', used to learn expected value in an introductory statistics course. The

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game is found to have enhanced pupils' understanding and retention, and fostered development of critical thinking skills. In a case study, Jančařík (2007) uses BlackJack game to show how it develops mathematical reasoning of future mathematics teachers. Littler and Jirotková (2004) explore in depth a game in which pupils should guess a solid or a geometric shape by asking yes-no questions. They carried out experiments with pupils of different ages and came to the conclusion that this game develops pupils' understanding of solids/shapes as well as their communicative skills which help them make precise and unambiguous statements about the solids and ask clear and precise questions. They also stress the diagnostic function of the game.

The focus of this paper is an educational game *Tandemat* used in teaching secondary mathematics.

2. Methodology

Tandemat (developed by Lucie Šilhánová) was inspired by the popular game Activity. In brief, three pairs of pupils compete in the game. Each team tries to get their figure to the last field of the board by correctly determining a mathematical concept which is written on a card: one of the team presents the concept within the time limit of 2 minutes, the other guesses. The cards have different colours according to the way the concepts written on them are presented (talking, drawing, pantomime) and are divided according to the number of points assigned to the concepts (1, 2 or 3). The board consists of a path with coloured fields – the figure moves for a number of points and the colour of the field where it ends determines the type of presentation for the team in the next round. The game is aimed for secondary pupils; however, the age of pupils depends on the choice of mathematical concepts (this is done by the teacher as well as the assignment of points).

The first version of the game was piloted with one group of pupils which resulted in the modification of the number of points for some concepts and of the way of presentation for others. The pilot study was also important for the experimenter to acquire experience in terms of the organisation of the game. Thus, this study concerns the adapted version of the game. The research questions were: What are the benefits and limits of Tandemat for the development of pupils' mathematical knowledge? What knowledge does the game bring to the teacher? Does the game have any benefits outside mathematics?

The game was trialled with 5 groups of pupils in 4 classes of a secondary grammar school: Group 1a (16 years old, 6 players), Group 1b (16 years old, 6 players), Group 2 (17 years old, 6 players), Group 3 (18years old, 5 players), Group 4 (19 years old, 6 players). Two to four rounds of the game were played in 50 to 75 minutes. The experiments were recorded by two video cameras (one aimed at the whole group, and one at pairs of pupils) and the transcriptions of verbal and nonverbal actions, pupils' written records and drawings, and experimenters' field notes were analysed using techniques of grounded theory approach (Strauss & Corbin, 1998).

Each unit of analysis (typically, one statement or several statements logically connected) was coded from the point of view of the person who presented the concept, the one who guessed the concept, and the others in the group. Example 1 includes a transcript of one situation where the codes are for the sake of clarity included in the description of the situation. Codes are given in square brackets, their dimension is written after the dash.

Example 1. Dana describes a 3-point concept "infinitely many solutions". Cilka is to guess it.
D1: *Zero x equals zero*. C2: *Hm*.
D3: *What is the root?* C4: *Zero? Or what? x? What?*
D5: *Zero x equals zero. What is the root?* C6: *What?* [Repeats D5] *Well, none. Or what, zero or what?*
D7: *What number can you substitute for x to get, when you multiply it by zero, to get zero?* C8: *Any.*
D9: [Gestures that it is so, only in different words.] C10: *So it is infinity, or what?*
D11: [Nods.] C12: *All as ... or what?*
D13: *Three words.* C14: *All real numbers. I don't know.*
D15: *The root is…? Or the root has…?* [Knocks three times on the desk.] C16: *Infinity solutions? Or what?*
D17: *So?* C19: *Well, the root has infinity…*

D20: *The root has infinitely…* [Says at the same time as Cilka.] C21: *Infinitely many solutions.*

Dana realises that infinitely many solutions can be the result of an equation, that is why she introduces an example of a simple linear equation [mathematical understanding – good, mathematical way of demonstration – correct, knowledge of the term – good]. She does not demonstrate words separately but as a phrase [fragmentation – none]. She knows that results of equations are called roots [mathematical terminology – correct]. As Cilka has problems guessing the concept, Dana gives her a hint via a logical question [specification of the concept]. Dana not only knows the term but also its content – she understands it.

Cilka knows the term of root and knows that she is to find the result. She starts guessing, but she is unsure [guessing – surface]. She offers wrong suggestions [association with example – wrong]. She answers D7 correctly. She already knows that the root of the equation is any real number [mathematical ideas] but she needs more help to correctly form the terms. At the end, she shows that she understands the concept [understanding the concept – good]. It may be that she might not have good understanding of equations as she suggests wrong words such as "zero in equations" [association – problems] or she might be stressed by the time limit. End of example.

3. Results

It has transpired in the analysis that the game can potentially help the teacher to diagnose pupils' mathematical understanding. Example 1 above shows this aspect of the game.

Example 2. Adam demonstrates a 3-point concept of "root of the equation" in words. Borek is to guess.

A1: *The first word – imagine a tree, it has branches, a trunk and in soil, it has?* [Gestures to suggest a tree.]

B2: *Root.* A3: *Yes. And we have been doing lately what?*

B4: *Equations. Root of the equation.* A5: *Yes.*

Adam demonstrates the concept in separate words [Fragmentation – in words]. Both words were demonstrated non-mathematically, and thus, we cannot diagnose his knowledge of the term and understanding its underlying concept [knowledge of the term – cannot be determined, mathematical understanding – cannot be determined]. We can infer that Borek at least knows the term [knowledge of the term – good]; however, we cannot say whether he has understanding of the concept [mathematical understanding – cannot be determined].

Example 3. Adam draws a 2-point concept of "factoring out". Borek is to guess it.

A1: [Begins to write *ab*+.] B2: *Polynomial? Some...*

A3: [Ads *ac*.] B4: *You subtract something?* A5: *No.*

A6: [Adds brackets around the expression, they resemble vertical lines; adds an arrow down.]

B7: *It is an absolute value.*

A8: *No. And ...* B9: [Points to the first line.] *Is this absolute value?*

A10: [Adds *a(b+c)* under the arrow.] B11: *It is factoring out.* A12: [Taps on the arrow.] *Yes!*

Adam uses a mathematical means of presentation, using a suitable binomial expression in symbolic form, and factors *a* out. He points to the relationship between the two expressions, how the second originated from the first. Adam understands the concept well. Borek is guessing as Adam is writing. He can understand the term polynomial, he at least knows the term absolute value and how it is recorded. He understands the concept of factoring out as he does not need Adam's hint in the form of tapping on the arrow. It is not clear why Borek suggests subtracting in B4.

The three examples represent both the pros and cons of diagnosis in the game. If the dialogue between the pupils continues for some time, it can provide the teacher with enough opportunities to uncover the possible understanding of the players. However, the concept can be guessed early or, as was the case in Example 2, it can be demonstrated in a non-mathematical way.

The second central category, c**onsolidation and development of mathematical competencies,** concerns the use of mathematics.

Example 4. Dana describes a 2-point concept of "square root". Cilka is to guess it.

D1: *X to the power of one half is?* C2: [Does not know, chuckles nervously.] *What? And which area is it from?*

D3: *It is from mathematics.* C4: *I see. From mathematics. But more specifically?*

D5: *Calculations. Algebra.* C6: *Something exponential? Or, I don't know.*

D7: *It's a relation between... anything. For example, nine has such a three. Four...* C8: *Power?*

D9: [Gestures that she is nearly there.] *But nine has three, not three has nine.* C10: *Root? Or what?*

D11: [Nods.] C12: *Root.*

D13: *And it's specific.* C14: *How specific?*

D15: *For example, that 81…9, and not 25 and 625.* C16: *What?* 625?

D17: *No, no.* 7 is…[Claps twice.] *of 49.* C18: *Root of two? Or what*?

D19: *No.* C20: *So only root. Or what?*

D21: *Which root?* C22: *What which root?*

D23: *What kinds of roots are?* C24: [Does not understand.]

D25: [Points to players who listed possibilities for their concept previously.] C26: *What? I could not hear it.*

D27: *7 is what ... from 49?* C28: *Square? Root of two? Root? Root of 49?*

D29: *No, root...* C30: *Root of 49?*

D31: *Yes, but which one?* C32: *Second?*

D33: *No.* C34: *Root? Second?* [...] C36: *Square root.*

Dana knows the concept and understands it well. She demonstrates it to Cilka in several ways. First, she mentions another record (D1) of the square root. She has to think about the area of mathematics where it belongs. Then she uses a concrete example (D7) which she specifies by a counter-example referring to the difference between the root and power. She uses small numbers and in order to lead Cilka towards the word second, she uses the term specific and then tries to find an example and counter-example for this term, too. In the example, numbers are 81 and 9, and in the counter-example, 25 and 625. It may be a mistake – we believe that she wanted to use 5 and 625. As Cilka still does not know, Dana uses a third way of demonstrating the relationship between the square root and power (D17). Dana consolidates the concept of square root and learns to communicate mathematically.

Cilka comes to realise that different types of roots exist and the one which she considers common (C20) is the square root. She practises using precise mathematical terminology and possibly realises its importance.

Mathematical thinking is developed, knowledge is consolidated when the player tries to include the concept in the appropriate area of mathematics and find relationships with other mathematical concepts, to find examples and counter-examples and characteristics of the concept which distinguish it from related concepts, to find the application of the concept both in mathematics and outside, to find characterisations of the concept from different points of view (if the team partner needs more help in guessing), to find different representations of the concept (definition, symbolisation, picture, etc.), to find appropriate questions to ask (in the case of the pupil who guesses).

Finally, the game develops competences other than mathematical, too. The players are motivated towards looking for synonyms, antonyms and using precise language. They are creative – they have to find different ways of expressing the same thing in drawing, words and pantomime. They have to interact with others in an efficient way. All the players in our experiments enjoyed playing the game (granted, they were volunteers).

4. Conclusions

Diagnosis of pupils' understanding is part of a teacher's everyday work. It can be done by usual techniques such as questions which he/she poses, by problem solving activities in the lessons or at home, by tests, etc. Some specific activities might be undertaken. Among them are diagnostic mathematics interviews (e.g., Moyer, Milewicz, 2002) and the deep analysis of pupils' written solutions (e.g., Schwarz, Wissmach & Kaiser, 2008). We have shown that Tandemat can be used for this purpose, too. Moreover, it can lead to consolidation of knowledge or even to new mathematical knowledge – both the demonstrator and the guesser can see mathematical concepts in new contexts, learn new properties, representations, etc. This is in agreement with Littler and Jirotková (2004) and their analysis of a game based on communication: "Our analysis shows that this need for communication helped linkages to be made in the pupils' minds between the isolated pieces of knowledge." (p. 66)

Three aspects of diagnosis via Tandemat are necessary to emphasise. First, it is usually diagnosis without any fear on the side of pupils. It is not a standard situation of assessment, so the pupils might be less afraid to show that they cannot understand something. Second, the diagnosis is not done by the teacher only, but by players themselves. The two players get immediate feedback on whether they understand the concept. By observing the game, the others at least revise mathematical terminology and think about the concept. On the other hand, a wrong demonstration of a concept can lead to a discussion resulting in re-education. The concept which has been forgotten comes to mind and some concepts are better understood. Third, the teacher can limit the concepts to only those which he/she wants to diagnose at the time – for example, at the beginning of a new topic, to see what pupils know about it, or at its end, to revise and see what needs more attention.

Nevertheless, there are some dangers to be taken into account. First, the time limit can be a stressing factor which influences the quality of pupils' demonstration or guessing. Thus, the diagnosis of their understanding must be done with caution. Second, the game is of limited value to mathematics when the concepts are demonstrated in non-mathematical ways. Some mathematical terms have their everyday connotations (see Example 2) and thus are prone to the non-mathematical treatment. Third, it may happen, even though we did not observe this (the mistake in D15 in Example 4 did not cause any problem), that a concept is demonstrated in a wrong way and the pupils appropriate this way. Thus, the teacher's role is important. He/she needs to observe the game carefully and use any such event for discussions. Fourth, during the game weak points of players are revealed and it can be stressing for some. Finally, some pupils may be shy to play the game.

As already stated, the teacher is a key agent for the game, not only as the organiser of the game and follow-up discussion, but also at the stage of preparation. The evaluation of chosen concepts in terms of points is a very difficult task. Some concepts may be considered difficult but may have a simple demonstration and vice versa.

The study has its limitation in the number of trials and also in the data. For our analysis, we only used the transcription of the game itself. It would have been better to speak to the pupils as well, for example, by showing the video to them and asking them what they meant, thought, etc. Unfortunately, at the time of our experiments it was not possible. On the other hand, we believe that the presented study shows that the game of Tandemat has potential benefits for the learning of mathematics at the secondary level.

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